

## ANALYSIS OF SKIN BURN INJURY THROUGH INTEGRAL TRANSFORM TECHNIQUES

### A. V. Presgrave

Instituto Militar de Engenharia - Seção de Engenharia Mecânica e de Materiais  
Praça General Tibúrcio 80 - 22290-270 Rio de Janeiro, RJ, Brazil  
amandapresgrave@gmail.com

### R. O. C. Guedes

Instituto Militar de Engenharia - Seção de Engenharia Mecânica e de Materiais  
Praça General Tibúrcio 80 - 22290-270 Rio de Janeiro, RJ, Brazil  
guedes@ime.eb.br

### F. Scofano Neto

Instituto Militar de Engenharia - Seção de Engenharia Mecânica e de Materiais  
Praça General Tibúrcio 80 - 22290-270 Rio de Janeiro, RJ, Brazil  
scofano@ime.eb.br

**Abstract:** *Skin burn injuries is one of the most common hazards encountered in daily life and in industrial environments such as petrochemical and plastic plants. These accidents are usually associated to contact with hot substances, gas leaks or intense heat fluxes associated to combustion processes. The main contribution of this work is to carefully assess the skin burn injury phenomenon by employing the well known Pennes bioheat equation. Here, the skin is taken as one layer medium subjected to a time varying heat flux boundary condition at its external surface. This model is handled analytically through means of integral transform techniques and the transient temperature field is studied in order to accurately predict the severity of the burn injury. The results are compared with previously reported data in the literature in order to discuss the relative merits of the mathematical model and the solution scheme presented in this contribution.*

**Keywords.** *Pennes equation, burn injury, integral transform.*

### 1. Introduction

Burn accidents are said to be one of most common and painful hazards a person may experience. Depending on the intensity of the burn, nerve endings may be severely damaged causing an intense distress and in some cases long-term hospitalization are required. Burn injuries can also affect muscles, bones, and blood vessels. Besides, a burn injury can impair the respiratory system and body temperature together with its thermal regulation.

Usually, burns are classified in two ways: the method of the burn and the degree of the burn, (Diller and Ryan; 1998). The most common causes of burn injuries related to the method are: thermal, chemical, electrical, light and radiation. As for the degree of the burn, the following assortment is found: first degree, second degree and third degree burns. A first degree burn is a superficial impairment that only affects the outer layer of the skin known as the epidermis. Usually this burn will heal on itself in a couple of days with minor or no scarring. In some cases, there may be peeling of the skin and some temporary discoloration. A second degree burn occurs when the injury affects the second layer of the skin - the dermal layer. The patient experiences deep intense pain and there may be some blisters together with some reddening of the skin. When treated with reasonable care, such burns will heal themselves in about three weeks time. A third degree burn is also referred as a full thickness burn since it affects all the layers of the human skin. Due to its nature, it is the most serious of all burns as it requires extensive medical care. Surprisingly, many third-degree burn patients do not report pain but this reaction is due to the fact that the nerve endings have been severely damaged.

A brief literature review suggests that the mathematical simulation of skin burns have received quite a lot of attention in the years following the end of the Second World War. More recent studies rely on the so-called Pennes' bioheat transfer equation. Pennes (1948) suggested that in order to account for the effect of the blood flow in a biological tissue, a source / sink term directly proportional to the difference between the temperature of the tissue and that of the arterial blood should be added in the standard heat diffusion equation. Torvi and Dale (1994) employed Pennes' bioheat transfer equation in order to predict skin temperatures and times for second and third degree burns under simulated flash fire conditions. Flash fires are hazards that are often encountered in petrochemical industries and are usually associated to intense heat fluxes of short duration, typically less than five seconds. By employing a finite element solution scheme in a three layered skin model, they performed a series of numerical investigations and compared their results with previously published results. Among their findings, they concluded that the wide variation of thermophysical properties mentioned by earlier investigators had minimum effect in the prediction of second degree burn and were found to be more relevant when a third degree injury was assessed. Also worth mentioning is the fact

that the blood perfusion term could be neglected in the determination of the transient temperature field. Liu et al (1999) also employed a one-dimensional analysis in a three layered skin by developing a thermal wave model of the bioheat transfer process. Since they were also interested in modeling flash fire situations, their main reason for utilizing this non-Fourier analysis was to estimate the deviations between the finite heat propagation velocity to that of the classical Pennes' model. They conducted a series of simulations in order to assess the role of the volumetric blood perfusion term and also found out that it could be neglected. They concluded that the mechanisms of wave like behavior of heat transfer in living tissues were complex and no generalization appeared to be possible at that point of their research. Their simulations suggested that only when an extremely high heat flux rate is present will the thermal wave effect dominate over the heat diffusion process. It appears that the main difficulty in established a conclusive idea relied on the fact that no well-established evaluation of the thermal relaxation time was available for biological tissues.

Ng and Chua (2002) studied the bioheat transfer equation for both the one-dimensional and two-dimensional situations by employing finite difference for the 1D case and a finite element package for the 2D simulation. A convective boundary condition and a constant temperature at the surface of the skin were utilized in order to simulate the heat source associated to the burn injury. They also attempted to estimate the effect the therapeutic efficacy of postburn cooling by simulating the immersion of the biological tissue in water at some selected temperatures once the heating period ended. Their main conclusion is that the one-dimensional model appeared to be quite accurate since the deviation for the estimates utilizing the 2D situation was quite small. Jian et al (2002) also employed a convective boundary condition at the outer layer of the human skin in order to predict the skin burn process. Their results suggest that the transient temperature field is significantly affected by the epidermis and dermis thicknesses while variations of the initial temperature and blood perfusion have little effect in temperature levels.

Mercer and Sidhu (2005) also utilized a one-dimensional multi-layer bioheat transfer model to study the effects of skin burn due to the deployment of automotive airbags. Again they found out that over the time scales of interest, typically around 0.5 to 2 seconds, the blood perfusion process had no major impact. Based on their numerical simulations, they concluded that a passenger may experience first and second degree burns due to venting of the airbags during deflation and also due to the direct contact with the fabric of the airbag.

## 2. Analysis

This section starts out by considering a generalized version of the heat transfer equation for a perfused organic tissue subjected to an external heat source in such a way in order to portray the skin burn injury problem. Here, we employ the well-known Pennes' equation model. As mentioned before, this model can be briefly described as a standard heat diffusion problem with an extra term that accounts for the blood flow in the organic tissue being analyzed. Therefore, the transient bioheat transfer problem is written as (Hartnett and Irvine, 1992):

$$\rho C \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + q_{met} + q_{ext} + q_{per} \quad (1)$$

The first term on the right hand side of Eq.(1) is immediately recognized as the heat diffusion term throughout the tissue. The next term,  $q_{met}$ , is the metabolic heat transfer rate per unit volume of tissue while  $q_{ext}$  represents the influence of an external heat source which, for example, can be relevant in the cases of hyperthermia treatment in cancerous tissues (Azevedo, 2004). The last term,  $q_{per}$ , is the heat transfer rate per unit volume of tissue due to blood perfusion. Based on his own experimental evidence, Pennes (1948) stated that the thermal impact of the blood flow could be characterized by introducing an energy sink term. This blood flow effect is assumed to be proportional to the volumetric perfusion level,  $\omega$ , and to the difference between the local tissue temperature and that of the arterial blood,  $(T - T_b)$  in such a way that:

$$q_{per} = -\rho_b C_b \omega (T - T_b) \quad (2)$$

Therefore, by considering the skin as a single layer, the transient one-dimensional heat transfer equation becomes (Presgrave, 2005; Presgrave et al. 2005):

$$\rho C \frac{\partial T(x,t)}{\partial t} = k \frac{\partial^2 T(x,t)}{\partial x^2} - \omega \rho_b C_b (T(x,t) - T_b) + q_{met}, \quad 0 < x < l, \quad t > 0 \quad (3)$$

$$T(x,0) = T_b \quad 0 \leq x \leq l \quad (4)$$

$$q_0 e^{-dt} + k \frac{\partial T(0,t)}{\partial x} = 0 \quad t > 0 \quad (5)$$

$$T(l,t) = T_b \quad t > 0 \quad (6)$$

An analysis of the above formulation shows that the skin is initially at the body core temperature,  $T_b$ , 37 °C, when suddenly an exponentially decaying heat flux is applied at the skin surface where  $d$  is the regression factor. Consistent with previous contributions, for example, Torvi and Dale (1994), it is also assumed that this heat flux basically affects the immediate vicinity of the skin surface and, consequently, the innermost layers remain at the body core temperature, Eq. (6), whose value is considered to be the same of that of the arterial blood.

The relations represented by Eqs. (3) to (6) can be written in a dimensionless form by employing the following variables.

$$\chi = \frac{x}{l} \quad (7)$$

$$\tau = \frac{k}{\rho C} \frac{t}{l^2} \quad (8)$$

$$\theta = \frac{T - T_b}{\frac{q_0 l}{k}} \quad (9)$$

Thus, it is a simple matter to show that the dimensionless version of the mathematical formulation being analyzed is expressed by:

$$\frac{\partial \theta(\chi, \tau)}{\partial \tau} = \frac{\partial^2 \theta(\chi, \tau)}{\partial \chi^2} - P_f \theta(\chi, \tau) + Q \quad 0 < \chi < 1, \quad \tau > 0 \quad (10)$$

$$\theta(\chi, 0) = 0 \quad 0 \leq \chi \leq 1 \quad (11)$$

$$e^{-\beta \tau} + \frac{\partial \theta(0, \tau)}{\partial \chi} = 0 \quad \tau > 0 \quad (12)$$

$$\theta(1, \tau) = 0, \quad \tau > 0 \quad (13)$$

where  $Q$ ,  $P_f$  and  $\beta$  are the dimensionless metabolic rate, perfusion coefficient and rate of decay of the external heat source which causes the burn injury. These quantities are expressed as follows:

$$Q = \frac{q_{met} l}{q_0} \quad (14)$$

$$P_f = \frac{\omega \rho_b C_b l^2}{k} \quad (15)$$

$$\beta = \frac{d \rho C l^2}{k} \quad (16)$$

A common assumption in burn injury studies, (Torvi and Dale, 1994; Liu et al, 1999, Jiang et al, 2002; Mercer and Sidhu, 2005), is to admit that both the perfusion and metabolic effects can be disregarded in the analysis since their contribution to the heat balance is usually much smaller than that of the external heat source. Pursuing this same idea, we seek the solution of problem (10) - (13) in terms of the following eigenfunction expansion:

$$\theta(\chi, \tau) = \sum_{i=1}^{\infty} A_i(\tau) \psi_i(\chi) \quad (17)$$

where the eigenfunctions  $\psi_i(\chi)$  are related to the Sturm-Liouville system described below:

$$\frac{d^2\psi_i(\chi)}{d\chi^2} + \mu_i^2 \psi_i(\chi) = 0 \quad 0 < \chi < 1 \quad (18)$$

$$\frac{d\psi_i(0)}{d\chi} = 0 \quad (19)$$

$$\psi_i(1) = 0 \quad (20)$$

Due to the simplicity of the above problem, its eigenfunctions are immediately recognized as  $\psi_i(\chi) = \cos(\mu_i \chi)$  and the norms are found to be  $N_i = \frac{1}{2}$ .

By employing the orthogonality property of the chosen eigenproblem, we conclude that:

$$A_i(\tau) = \frac{1}{N_i} \int_0^1 \theta(\chi, \tau) \psi_i(\chi) d\chi \quad (21)$$

and thus the integral-transform pair is found to be (Cotta; 1993, Cotta; 1998):

$$\bar{\theta}_i(\tau) = \frac{1}{N_i^{1/2}} \int_0^1 \theta(\chi, \tau) \psi_i(\chi) d\chi \quad \text{transform relation} \quad (22)$$

$$\theta(\chi, \tau) = \sum_{i=1}^{\infty} \frac{1}{N_i^{1/2}} \psi_i(\chi) \bar{\theta}_i(\tau) \quad \text{inverse relation} \quad (23)$$

The next step is to rewrite the original problem formulation in terms of the transformed variable  $\bar{\theta}_i(\tau)$ . This task is accomplished through a series of mathematical steps which are well-documented in Mikhailov and Ozisik (1984). The resulting decoupled system of ordinary differential equations that govern  $\bar{\theta}_i(\tau)$  is:

$$\frac{d\bar{\theta}_i(\tau)}{d\tau} + \mu_i^2 \bar{\theta}_i(\tau) = \frac{1}{N_i^{1/2}} \psi_i(0) e^{-\beta\tau} \quad (24)$$

while its initial conditions is determined by employing the integral transform, Eq. (22), in relation (11) to yield:

$$\bar{\theta}_i(0) = 0 \quad (25)$$

System (24) is solved analytically to obtain:

$$\bar{\theta}_i(\tau) = \frac{1}{N_i^{1/2}} \frac{\psi_i(0)}{\mu_i^2 - \beta} \left( e^{-\beta\tau} - e^{-\mu_i^2 \tau} \right) \quad (26)$$

and by inserting this result in the inverse relation, Eq. (23), the dimensionless temperature field is expressed as:

$$\theta(\chi, \tau) = 2 \sum_{i=1}^{\infty} \frac{\cos(\mu_i \chi)}{\mu_i^2 - \beta} + 2 \sum_{i=1}^{\infty} \frac{\cos(\mu_i \chi)}{\beta - \mu_i^2} e^{-\mu_i^2 \tau} \quad (27)$$

While Eq. (27) is indeed a closed form solution for the burn injury formulation here analyzed, the convergence characteristics of its first term are expected to be poor since the chosen eigenvalue problem does not account for the non-homogeneous term associated to the external heat flux. Thus, we seek an alternative solution based on the “split-

up” solution procedure similar to those discussed in Mikhailov and Ozisik (1984). The basic idea is to consider the temperature field as the sum of two contributions which are expressed by an auxiliary problem,  $\theta_{aux}(\chi)$ , and a homogeneous problem  $\theta_h(\chi, \tau)$  in such a way that:

$$\theta(\chi, \tau) = \theta_{aux}(\chi)e^{-\beta\tau} + \theta_h(\chi, \tau) \quad (28)$$

where the problem for  $\theta_{aux}(\chi)$  is given by:

$$\frac{d^2\theta_{aux}(\chi)}{d\chi^2} + \beta\theta_{aux}(\chi) = 0 \quad 0 < \chi < 1 \quad (29)$$

$$\frac{d\theta_{aux}(0)}{d\chi} + 1 = 0 \quad (30)$$

$$\theta_{aux}(1) = 0 \quad (31)$$

By inserting Eq. (28) in relations (10) - (13) and with the aid of problem (29) - (31), we find that  $\theta_h(\chi, \tau)$  is governed by:

$$\frac{\partial\theta_h(\chi, \tau)}{\partial\tau} = \frac{\partial^2\theta_h(\chi, \tau)}{\partial\chi^2} \quad 0 < \chi < 1, \tau > 0 \quad (32)$$

$$\theta_h(\chi, 0) = -\theta_{aux}(\chi) \quad 0 \leq \chi \leq 1 \quad (33)$$

$$\frac{\partial\theta_h(0, \tau)}{\partial\chi} = 0 \quad \tau > 0 \quad (34)$$

$$\theta_h(1, \tau) = 0 \quad \tau > 0 \quad (35)$$

The above problem can be readily solved by the same eigenfunction expansion procedure described earlier and consequently the transient temperature field is:

$$\theta_h(\chi, \tau) = -\sum_{i=1}^{\infty} \left\{ \frac{1}{N_i} \int_0^1 \theta_{aux}(\chi) \psi_i(\chi) d\chi \right\} \psi_i(\chi) e^{-\mu_i^2 \tau} \quad (36)$$

where the solution for the  $\theta_{aux}(\chi)$  problem is given by:

$$\theta_{aux}(\chi) = \frac{\tan(\sqrt{\beta})}{\sqrt{\beta}} \cos(\sqrt{\beta}\chi) - \frac{\sin(\sqrt{\beta}\chi)}{\sqrt{\beta}} \quad (37)$$

Accordingly, an alternative expression for the dimensionless skin burn injury problem formulation with enhanced convergence characteristics is :

$$\theta(\chi, \tau) = \frac{e^{-\beta\tau}}{\sqrt{\beta}} \left\{ \tan(\sqrt{\beta}) \cos(\sqrt{\beta}\chi) - \sin(\sqrt{\beta}\chi) \right\} + 2 \sum_{i=1}^{\infty} \frac{\cos(\mu_i\chi)}{\beta - \mu_i^2} e^{-\mu_i^2 \tau} \quad (38)$$

where the eigenvalues  $\mu_i$  are determined from:

$$\mu_i = \frac{(2i-1)\pi}{2} \quad (39)$$

### 3. Results and Discussion

Having established two analytical solutions for the skin burn injury problem, Eq. (27) and (38), we are now in a position to evaluate the relative merits of the procedures outlined in the previous section and also to establish some insight into the physical problem in question. However, it seems naturally reasonable to initially address the role of both the perfusion and the metabolic heat effects in the heat transfer process.

Table 1 - Geometry and Properties of the Skin

	Specific Heat $C \left[ \frac{J}{kg \cdot ^\circ C} \right]$	Blood Perfusion Rate $\omega \left[ \frac{m^3 / s}{m^3} \right]$	Thermal Conductivity $K \left[ \frac{W}{m \cdot ^\circ C} \right]$	Thickness $l [m]$	Density $\rho \left[ \frac{kg}{m^3} \right]$
<b>Epidermis</b>	3578 - 3600	0	0.21 - 0.26	$80 \times 10^{-6}$	1200
<b>Dermis</b>	3200 - 3400	0.00125	0.37 - 0.52	0.00200	1200
<b>Sub-Cutaneous</b>	2288 - 3060	0.00125	0.16 - 0.21	0.01000	1000
<b>Blood</b>	3770	-----	-----	-----	1060
<b>Single-Layer (in-vivo)</b>	3600	0.00125	0.48 - 2.80	0.01208	1200
<b>Single-Layer (in-vitro)</b>	3600	0.00125	0.21 - 0.41	0.01208	1200

Table 1 shows some relevant thermophysical properties of the three layers of the human skin, namely the epidermis, the dermis and the sub-cutaneous tissues. Also presented are the properties for the single layer human skin collected from both in-vivo and in-vitro experiments (Torvi and Dale, 1994). The rate of metabolic energy production is usually between 100 and 300  $W/m^2$ . Although many scenarios for the burn injury problem can be envisioned, here we are basically interested in the so-called “flash-fire” accident. These situations are associated to high heat fluxes, typically in the range of 24 to 84  $kW/m^2$  with an exposure time of about 3 to 5 seconds. For these specific cases, the regression coefficient  $d$  is around unity since the ratio  $\frac{q}{q_0}$  is less than 1% at the end of a 5 second exposure. Therefore,

characteristic values of the dimensionless metabolic heat source, perfusion coefficient and regression coefficient are found to be around  $1.5 \times 10^{-4}$ , 4.5 and 3900. Clearly, it seems reasonable to assume that the contribution of the external source is the most dominant effect in the heat transfer process. Of course, care should be taken in this conclusion since this source is only active for about 5 seconds and there might be some speculation about the role of both the perfusion process and metabolic heat once the heat source extinguishes. As mentioned before, the literature review suggests that these effects are only relevant in situations where a low heat flux is applied over a reasonably long exposure time. Moreover, it takes about 20 seconds for the skin to react to the thermal load by increasing blood flow in the affected areas (Torvi and Dale, 1994) and in the presence of high fluxes, second and third degree burns are much likely to develop before 20 seconds. In addition to this reasoning, numerical multi-layered skin simulations such as those of Liu et al. (1999), corroborate the fact that blood perfusion effects and metabolic heat production have minimum importance in the simulation of the transient temperature distributions.

Another important issue is the convergence rate of both the classical integral transform solution, Eq. (27), and of the split-up solution, Eq. (38). In general terms, it was found that a truncation order of  $N=100$  is adequate enough to warrant graphical convergence for the split-up solution at times greater than  $10^{-2}$  seconds. As previously anticipated, numerical simulations revealed that the direct application of the integral transform procedure to the skin burn formulation resulted in very poor convergence patterns. Perhaps, this conclusion is better envision by inspecting figures 1 and 2 which present the transient temperature field for both the two solution schemes for the case of an initial incident flux of  $q_0 = 54 \frac{kW}{m^2}$  and an exposure time of 5 seconds. The transient temperature distribution study for the skin surface (fig. 1) and the basal layer (fig. 2), which marks the transitional point between the epidermis and the dermis, show that a 500 expansion term for direct approach presents a strong deviation from the more accurate solution based on the split-up procedure, during the heating phase which lasts for about 1 second. As time progresses, both figures 1 and 2 indicate that the two solution schemes yield identical results. As a matter of fact, this trend can be

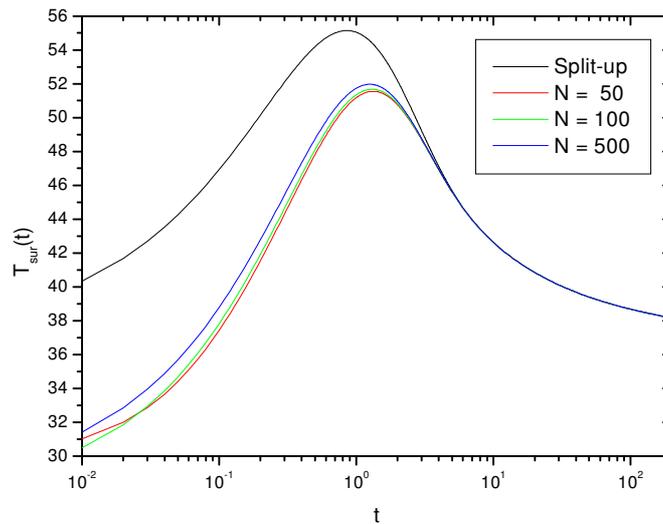


Figure 1. Converged Split-up Solution versus Direct Solution  $k = 0.764 \text{ W/m}^\circ\text{C}$ ,  $q_0 = 54 \text{ kW/m}^2$ ,  $d = 1(\text{l/s})$

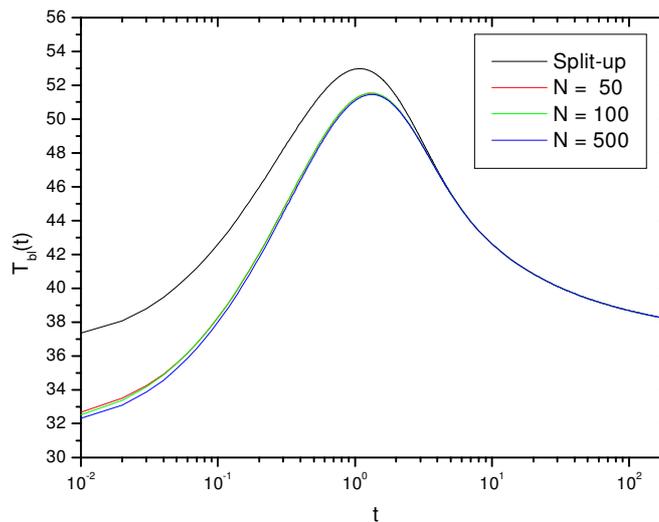


Figure 2. Converged Split-up Solution versus Direct Solution  $k = 0.764 \text{ W/m}^\circ\text{C}$ ,  $q_0 = 54 \text{ kW/m}^2$ ,  $d = 1(\text{l/s})$

explained upon an inspection of the chosen eigenvalue problem, Eq. (18) – (20). The boundary condition of the Sturm-Liouville problem at the skin surface, Eq. (19), cannot take into account the influence of external heat source and therefore, it is no surprise to find a significant deviation between the two solutions during the early transient stages. On the other hand, as time increases, the external heat source rapidly decays due to its exponential nature and consequently relation (19) becomes a more accurate representation of the physical problem which is expressed by a match of the two solution schemes. Our findings suggest that the direct approach only yields good quality results for the latter part of the transient process and does not capture adequately the more important heating phase. As a general rule, solution (27) should be discarded in favor of the split up procedure and accordingly the next simulations are based only through the evaluation of Eq. (38).

Figure 3 presents the transient temperature field at three selected locations, namely the skin surface ( $x = 0\mu\text{m}$ ), the basal layer ( $x = 80\mu\text{m}$ ) and at the mid plane of the epidermis ( $x = 40\mu\text{m}$ ). An examination of these results shows that, during the heating phase, the skin surface reaches a peak temperature of about  $55^\circ\text{C}$  while the basal layer seems to be  $2^\circ\text{C}$  colder. Throughout the cooling phase, the temperature fields at all the three positions rapidly collapses and decay at the same rate since the external heat source is no longer in action. Precise evaluations of the transient temperature field,

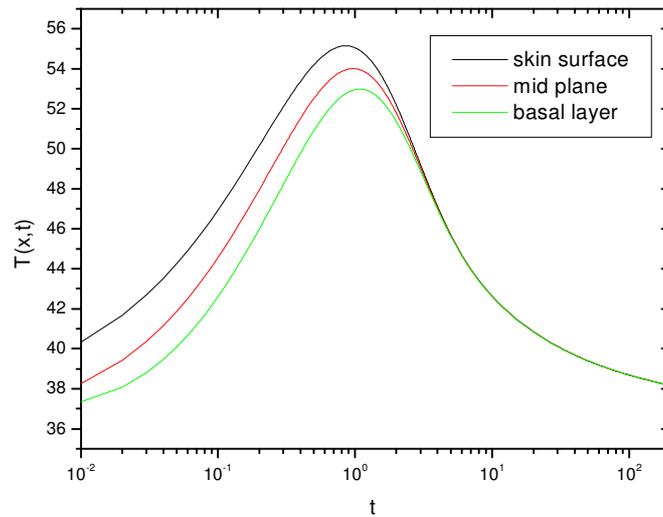


Figure 3. Transient Temperature Distributions  $k = 0.764 \text{ W/m}^\circ\text{C}$ ,  $q_0 = 54 \text{ kW/m}^2$ ,  $d = 1(1/s)$

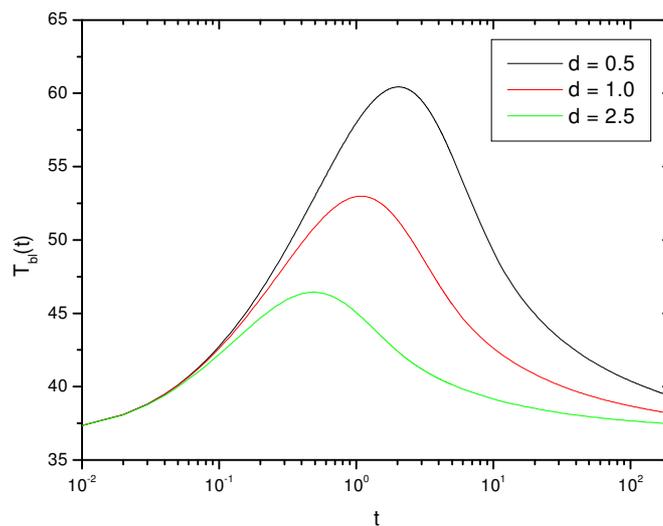


Figure 4. Basal Layer Temperature Distributions  $k = 0.764 \text{ W/m}^\circ\text{C}$ ,  $q_0 = 54 \text{ kW/m}^2$

especially at the basal layer, are important since thermal damage occurs once the skin temperature reaches  $44^\circ\text{C}$  (Torvi and Dale, 1994). This particular simulation, related to a five second exposure time for an initial heat flux of  $q_0 = 54 \frac{\text{kW}}{\text{m}^2}$ , shows that the basal layer is above  $44^\circ\text{C}$  during approximately 10 seconds and thus some thermal damage, probably a first or even a second degree burn, is expected.

Figures 4 and 5 study the influence of the exposure time in the transient temperature field for both the skin surface and the basal layer considering a fixed initial heat flux of  $q_0 = 54 \frac{\text{kW}}{\text{m}^2}$ . As expected, long exposure times such 10 and 5 seconds ( $d = 0.5$  and  $1.0$ , respectively) will result in high peaks of temperatures as displayed in figs. 4 and 5. The curve associated to  $d = 2.5$  ( exposure time of 2 seconds) is an application related to a possible thermal injure on a driver's hands due to the venting of hot gases from airbag depletion in car accidents. An inspection of both figures 4 and 5 to

this particular case shows that minimum thermal damage is expected since the temperatures levels are slightly above the 44°C threshold and for a short time of less than one second. This observation is consistent with the findings of Mercer and Sidhu (2005) whose simulations predicted that first degree burn injuries will only develop under extreme conditions.

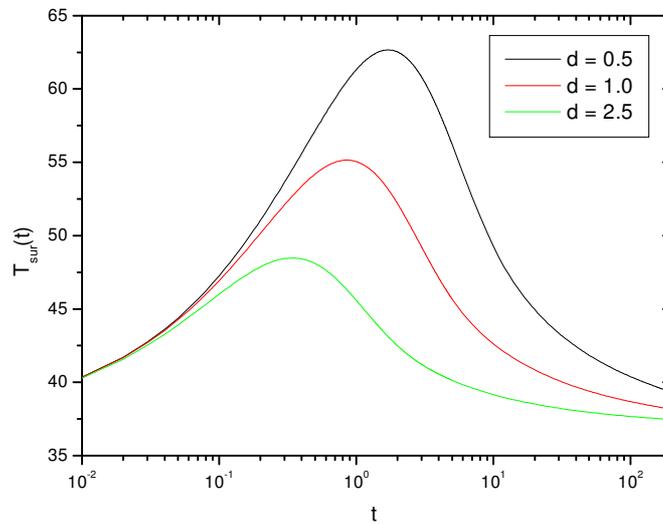


Figure 5. Skin Surface Temperature Distributions  $k = 0.764 \text{ W/m}^\circ\text{C}$ ,  $q_0 = 54 \text{ kW/m}^2$

Finally, the variations of the initial heat flux are assessed in figs. 6 and 7 for a fixed exposure time of 5 seconds. The case related to the case of  $q_0 = 84 \frac{\text{kW}}{\text{m}^2}$  is of particular interest since it corresponds to a typical exposure of propane gas flash fire on nude skin (Torvi and Dale, 1994). The simulations in fig. 7 reveal that the temperature at the basal layer reaches a peak of about 60 °C at approximately one second after the initial exposure. Also, this layer remains above the 44°C threshold for about 10 seconds suggesting that a severe burn injury will most likely occur. On the other hand, the other two cases might represent the heat flux incident on skin from such a fire when covered with a protective garment. It is interesting to notice that in the situation related to  $q_0 = 24 \frac{\text{kW}}{\text{m}^2}$  no thermal damage is expected since the temperature levels are below 44°C.

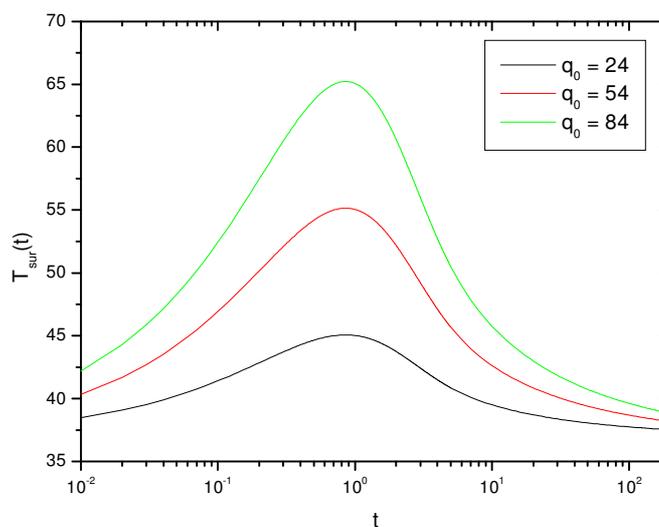


Figure 6. Skin Surface Temperature Distributions  $k = 0.764 \text{ W/m}^\circ\text{C}$ ,  $d = 1(1/\text{s})$

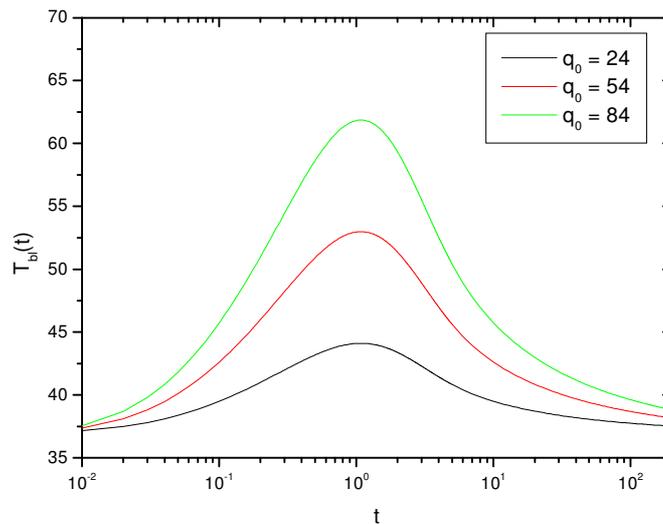


Figure 7. Basal Layer Temperature Distributions  $k = 0.764 \text{ W/m}^\circ\text{C}$ ,  $d = 1(1/\text{s})$

#### 4. Conclusion

This contribution advanced two solution schemes based on integral transform techniques in order to address the skin burn injury problem modeled by the bioheat transfer equation. Based on our simulations, the direct application of the integral transform technique yielded poor convergence characteristics during the heating phase and did not prove to be a good solution procedure to the problem. On the other hand, the split-up procedure provided fast convergence rates and is recommended in the simulations of thermal damages on human skin due to an external heat source. The simulations also revealed that the transient temperature fields are quite sensible to variations of exposure time and to the intensity of the surface heat flux. Our current research aims at characterizing the burn injury in a more precise way by employing the cumulative integral rate of tissue damage and by considering the simultaneous effects of all the three skin layers.

#### 5. References

- Azevedo, M. D. B., 2004, "Analytical Numerical Simulation of the Bioheat Transfer in Organic Tissues" (in Portuguese), M.Sc. Dissertation, IME, Rio de Janeiro, Brazil, 252 p.
- Cotta, R. M., 1993, "Integral Transforms in Computational Heat and Fluid Flow", CRC Press, Florida, 340 p.
- Cotta, R. M. (ed.), 1998, "The Integral Transform Method in Thermal and Fluids Science and Engineering", Begell House, New York, 430 p.
- Diller, K., R. and Ryan, T. P., 1998, "Heat Transfer in Living Systems: Current Opportunities", Journal of Heat Transfer, Vol. 120, pp. 810-829.
- Hartnett, J.P. and Irvine, T.F. (eds.), 1992, "Advances in Heat Transfer", Academic press, Vol. 22, 586 p.
- Jiang, S. C., Ma, N. and Zhang, X. X., 2002, "Effects of Thermal Properties and Geometrical Dimensions on Skin Burn Injuries", Burns, Vol. 28, pp. 713-717.
- Liu, J., Chen, X. and Xu, L.X., 1999, "New Thermal Wave Aspects on Burn Evaluation of Skin Subjected to Instantaneous Heating", IEEE Transactions on Biomedical Engineering, Vol. 46, No. 4, pp. 420-428.
- Mercer, G. N. and Sidhu, H.S., 2005, "Modeling Thermal Burns due to Airbag Deployment", Burns, Vol. 31, pp. 977-980.
- Mikhailov, M. D. and Özisik, M. N., 1984, "Unified Analysis and Solutions of Heat and Mass Diffusion", Dover Publications, New York, 458 p.
- Ng, E.Y.K. and Chua, L.T., 2002, "Comparison of one- and two-dimensional Programmes for Predicting the State of Skin Burns", Burns, Vol. 28, pp. 27-34.
- Pennes, H.H., 1948, "Analysis of Tissue and Arterial Blood Temperatures in the Resting Human Forearm", Journal of Applied Physiology, Vol. 1, pp. 93-122.
- Presgrave, A. V., 2005, "Modelling and Simulation of Blood Perfusion Effects in Bioheat Transfer Problems" (in Portuguese), M.Sc. Dissertation, IME, Rio de Janeiro, Brazil, 205 p.
- Presgrave, A. V., Guedes, R. O. C. and Scofano Neto, F., 2005, "Hybrid Analytical Numerical Solution to the Bioheat Transfer Equation", Proceedings of the 18th International Congress of Mechanical Engineering, Ouro Preto, Brazil.

Torvi, D. A. and Dale, J. D., 1994, "A Finite Element Model of Skin Subjected to a Flash Fire", *Journal of Biomedical Engineering*, Vol. 116, pp. 250-255.

## **6. Copyright Notice**

The authors are the only responsible for the printed material included in this paper.